The Right Chemistry for Synchrony and Asynchrony

or, when is a chemical abstract machine implementable?

Sanjiva Prasad
IIT Delhi

joint work with Deepak Garg and Akash Lal

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The Right Chemistry for Synchrony and Asynchrony

Talk Overview

About the Wadleresque title of the talk...

- Overview of the Chemical Metaphor
- Motivation
- Oriented Rewriting Theory and all that
- Asynchrony: TCCS CHAM
- Synchrony: SCCS CHAM
- Conclusions, further work
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The Chemical Metaphor


- Program components = **Molecules** \( m_i \)
- Molecules float around in a **Solution**: \( \mathcal{S} = \{ m_1, \ldots, m_k \} \)
- **Magical Mixing**: molecules in a solution unordered.
- Interaction = **Reaction**. (Nondeterministic)
  
  Rules of the form \( \{ m_1, \ldots, m_k \} \mapsto \{ m'_1, \ldots, m'_l \} \).

- Agglomerate molecules **ionize** in a solution \( \{ p_1 | p_2 \} \leftarrow \{ p_1, p_2 \} \).
- Structure of molecules adjusted to facilitate reaction: **isomers**

Molecules can be complex – nested solutions possible.
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Example: a simple process calculus

Names: $x \in \mathcal{N}$  
Actions: $\alpha \in \mathcal{Act} = \mathcal{N} \cup \overline{\mathcal{N}}$

Processes: $p ::= 0 \mid \alpha.p \mid \nu x.p \mid p|p$

Molecules: $m ::= p \mid \nu x.S$

Solutions: $\mathcal{S} ::= \{m_1, \ldots, m_k\} \ (k \geq 0)$

Contexts $\mathcal{C} ::= [\ ] \mid \mathcal{C} \uplus \mathcal{S} \mid \{\nu x.C\}$

A solution with a “solution-shaped hole” in it.
Example contd.. Rules

Rules: reaction, heating-cooling (reversible), clean-up

\[
\begin{align*}
\{x.p, \overline{x}.q\} & \Rightarrow \{p, q\} \quad (R) \\
\{p|q\} & \Rightarrow \{p, q\} \quad (P) \\
\{\nu x.p\} & \Rightarrow \{\nu x.\{p\}\} \quad (M) \\
\{\nu x.S, p\} & \Rightarrow \{\nu x.\{p\} \cup S\} \quad x \notin f v(p) \quad (E) \\
\{0\} & \sim \{\} \quad (0c) \\
\{\nu x.S\} & \sim S \quad x \notin f v(S) \quad (\nu c)
\end{align*}
\]

Applicable in any context specified earlier.
SOS versus CHAM

Process calculus term describing 3 processes in parallel: \((a.b.0|\overline{b}.0)|\overline{a}.0\)

SOS: syntax-oriented inference rules to show that a program component has an interaction capability. Tree structured proofs.

\[
\begin{align*}
(a.b.0 & \rightarrow^a b.0) \\
(a.b.0|\overline{b}.0) & \rightarrow^a (b.0|\overline{b}.0) \\
\overline{a}.0 & \rightarrow^{\overline{a}} 0 \\
(a.b.0|\overline{b}.0)|\overline{a}.0 & \rightarrow^\tau (b.0|\overline{b}.0)|0
\end{align*}
\]

(1)

\[
\begin{align*}
b.0 & \rightarrow^b 0 \\
\overline{b}.0 & \rightarrow^{\overline{b}} 0 \\
b.0|\overline{b}.0 & \rightarrow^\tau 0|0 \\
(b.0|\overline{b}.0)|0 & \rightarrow^\tau (0|0)|0
\end{align*}
\]

(2)

CHAM style:

\[
\{[(a.b.0|\overline{b}.0)|\overline{a}.0]\} \rightarrow^* \{a.b.0, \overline{b}.0, \overline{a}.0]\} \leftrightarrow \{b.0, \overline{b}.0, 0]\} \leftrightarrow \{0, 0, 0\}
\]
Motivations and Advantages

- Eliminates the distributed redex problem
  Brings together distant parts by structural adjustment.
- Local rewriting, not tree-structured proofs
  conquers the bureaucratic rigidity of syntax
- Supports formulations with reduction, not labelled transitions.
- Concurrency, concurrency, concurrency (rewriting in contexts)

CHAM theory motivated by implementation considerations.

Prototypical ideas on mobility (transcending structure) and locality (membranes).
The Counter-Reformation of SOS

Milner (1990) immediately recognized the benefits of the CHAM approach.

- Define a notion of structural equivalence $\equiv$
- Provide transition rules as before.
  - But fewer cases – if $p \mid q \equiv q \mid p$, then don’t need the symmetric case of the following rule:

$\begin{align*}
  p \rightarrow^{\alpha} p' \\
  p \mid q \rightarrow^{\alpha} p' \mid q
\end{align*} \quad \text{PAR}$

- Include the STRUCT rule:

$\begin{align*}
  p \equiv p' \\
  p \rightarrow q \\
  q \equiv q'
\end{align*} \quad \text{STRUCT}$

Transitions now on equivalence classes modulo $\equiv$. 
Motivation
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**Reasonable notions of structural equivalence**

The STRUCT rule is not *effective* for implementation:

- No guide to picking appropriate element $p$ of $[p']\equiv$ that can react.
- No bounds on amount of transformation from $p'$ to $p$, or what transformations to do.

Central question:

*When is the “$\equiv$” relation *reasonable*?*

Answer 1: $p \equiv p'$ should be *decidable*,
and ....

Milner: “structural laws should be digestible without concern for the dynamics of actions”

...anything further? Few formal guidelines.
Approach: Revisit CHAM Theory

Classification of Rules in CHAMs

Rules specific to a calculus

- **Reaction Rules**: $\{m_1, \ldots, m_k\} \mapsto \{m'_1, \ldots, m'_l\}$

- **Structural Rules**: Heating-Cooling: $S \Rightarrow S'$ and Clean up: $S \rightsquigarrow S'$

General Laws:

- **Chemical Law** — rewrite freely in a subsolution

$$
S \rightarrow S' \\
S \cup S'' \rightarrow S' \cup S''
$$

- **Membrane Law** — rewrite within contexts

$$
S \rightarrow S' \\
\{C[S]\} \rightarrow \{C[S']\}
$$

- **Airlock Law**: $\{m, m_1, \ldots, m_n\} \Leftarrow \{m \triangleleft \{m_1, \ldots, m_n\}\}$
CHAMs implementable, given the Right Orientation

Intuition: Rules are “naturally oriented”

- Heating $\rightarrow$ increases reactivity
- Cooling is its inverse, doesn’t contribute to reaction ($\rightarrow = \rightarrow^{-1}$)
- Clean-up $\sim$ gets rid of inert molecules, doesn’t reduce reactivity
- Reactions are not conditional on absence of molecules (cf Chemical Law)

Let $\rightarrow \mathcal{A} = \rightarrow \cup \sim$ and let $=\mathcal{A}$ be its symmetric, reflexive transitive closure.

Implementation idea: Heat and clean-up as much as possible before reaction; postpone cooling to the end.

But is the oriented version equivalent to the specified semantics of $[p] \equiv \leftrightarrow [q] \equiv$?

Yes, under certain conditions on $\rightarrow \mathcal{A}$, $\leftrightarrow$. 
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**Operational Effectiveness**

\[ \rightarrow_\mathcal{A} \text{ and } \rightarrow_\mathcal{R} \] should satisfy the following properties.

**Confluence**

\[ \text{S' } \rightarrow_\mathcal{A} \text{ S'', S' } \rightarrow_\mathcal{A} \text{ S', S' } \rightarrow_\mathcal{A} \text{ S''} \]

**Coherence**

\[ \text{S } \rightarrow_\mathcal{A} \text{ S', S } \rightarrow_\mathcal{A} \text{ S1, S } \rightarrow_\mathcal{A} \text{ S2, S } \rightarrow_\mathcal{A} \text{ S3, S } \rightarrow_\mathcal{A} \text{ S'} \]

**Theorem (Standardization).** If a set of oriented structural rules is operationally effective then \( \forall n \geq 0, \)

\[ (\rightarrow_\mathcal{A}; \rightarrow_\mathcal{R}; \rightarrow_\mathcal{A})^n \subseteq \rightarrow_\mathcal{A}; (\rightarrow_\mathcal{R}; \rightarrow_\mathcal{A})^n; (\rightarrow_\mathcal{A}^{-1})^n \]
If $\rightarrow_\mathcal{A}$ is **terminating**: naive implementation is correct

Implementation complete w.r.t. Specification
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Getting really nit-picky

Of course, all of these rewritings are modulo an Equational Theory $\mathcal{E}$.

When $\mathcal{E} = AC$ then coherence is verifiable. While NP-complete in general, if the rules are left linear then it is tractable. Monoidal identity and idempotence in general cause non-termination, but harmless in CHAMs due to constraints on clean-up rules.

We identify configurations upto

- Change of bound variables
- Reordering of restriction. But a reformulation and factoring of restriction rules makes it ’somehow” seem an AC theory.
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Full Abstraction with LTS

Let $\sim_p$ and $\sim_c$ stand for bisimulation equivalence on $\mathcal{P}$ (process calculus) and $\mathcal{C}$ (CHAM)

Translation $\langle . \rangle : \mathcal{P} \rightarrow \mathcal{C}$

- Adequate: $\langle p \rangle \sim_c \langle q \rangle$ implies $p \sim_p q$
- Fully Abstract: $\langle p \rangle \sim_c \langle q \rangle$ iff $p \sim_p q$

Full abstraction is a typical notion of “correctness” of $\mathcal{C}$ vis a vis $\mathcal{P}$. 
Effectiveness in proving full abstraction

**Theorem**: If $C$ is operationally effective and both forward and backward simulates $P$, then it is **fully abstract** (w.r.t. bisimulation on $P$).

Idea: Define bisimulations using equivalence classes of $=_{A}$. 

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**CHAM $C$ forward simulates** $P$ if $\forall p, q \in P$ s.t. $p \rightarrow^{\alpha} q$

$\exists S \text{ s.t. } \langle p \rangle \Rightarrow^{\alpha} S =_{A} \langle q \rangle$.

**CHAM $C$ backward simulates** $P$ if $\forall p, S$, s.t. $\langle p \rangle \Rightarrow^{\alpha} S$,

$\exists p' \text{ s.t. } \langle p' \rangle =_{A} S \text{ and } p \rightarrow^{\alpha} p'$. 

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**TCCS:** ccs without \( \tau \)

\[
p ::= 0 \mid \alpha.p \mid \nu x.p \mid p|p \mid p|p \mid \text{fix}_i(\vec{x} = \vec{p}) \mid \ldots
\]

\[
\begin{array}{ll}
\alpha.p \rightarrow^\alpha p & p_1 \rightarrow^\alpha p_1' \\
p_1 \rightarrow p_1' & p_1 \rightarrow^\alpha p_1' \\
p_1 | p_2 \rightarrow p_1' | p_2 & p_1 \rightarrow^\alpha p_1' \\
p_1 \rightarrow p_1' & p_1 \rightarrow^\alpha p_1' \\
p_1 | p_2 \rightarrow p_1' | p_2 & p_1 \rightarrow^\alpha p_1' \\
p_1 \rightarrow p_1' & p_1 \rightarrow^\alpha p_1' \\
\nu x.p \rightarrow \nu x.p' & \nu x.p \rightarrow^\alpha \nu x.p'
\end{array}
\]

\[
\begin{array}{ll}
p_1 \rightarrow^\alpha p_1' & p_2 \rightarrow^\alpha p_2' \\
p_1 | p_2 \rightarrow p_1' | p_2' & p_2 \rightarrow^\alpha p_2' \\
p_1 \rightarrow^\alpha p_1' & p_1 \rightarrow^\alpha p_1' \\
p_1 | p_2 \rightarrow p_1' | p_2' & p_2 \rightarrow^\alpha p_2' \\
p_1 \rightarrow^\alpha p_1' & p_1 \rightarrow^\alpha p_1' \\
p_1 \rightarrow p_1' & p \rightarrow^\alpha p'
\end{array}
\]

\[\alpha \notin \{x, \bar{x}\}\]
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The Trouble with Berry-Boudol’s TCCS CHAM

It’s not operationally effective!
Airlock makes Heating not confluent.
Cooling actually increases reaction potential.
Consider:

\[\{\nu b. (a \cdot p \mid r) \mid \overline{a}.q\} \rightarrow^* \{\nu b. \{a \cdot p, r\}, \overline{a}.q\}\]

(Airlock) \[\rightarrow \{\nu b. \{a \cdot p \triangleleft \{r\}\}, \overline{a}.q\}\]

(HeavyIon) \[\rightarrow \{\nu b. \{a \cdot (p \triangleleft \{r\})\}, \overline{a}.q\}\]

(RestrMemb) \[\rightarrow \{\nu b. (a \cdot (p \triangleleft \{r\}))\}, \overline{a}.q\}\]

(RestrIon) \[\rightarrow \{a.(\nu b. (p \triangleleft \{r\}))\}, \overline{a}.q\}\]

(React) \[\rightarrow \{\nu b. (p \triangleleft \{r\}), q\}\]

\[\rightarrow^* \{\nu b. (p \mid r) \mid q\}\]

Some other minor problems – definition of Contexts for Membrane law
**New TCCS CHAM**

Problem: External Choice uses Airlock in Berry-Boudol CHAM. Treatment not compositional w.r.t external choice.

**Tags**: Sets of boolean literals. Tags of mutually exclusive choices differ on a literal. Choice implemented by “Speculative Concurrent Execution”.

**Tag management** (“catalysis?”): Keep track of active tags – private $A$ and public $L$

Molecule with inactive tags is garbage, can’t execute.

*Reaction* removes unchosen tags from active set (a choice is made)

**Configurations**: $A \vdash^L S$

**Molecules**: $m ::= p^T \mid \nu x.S$

**Contexts**: $C ::= [\ ] \mid C \uplus S \mid \{\nu x.C\}$
**Some TCCS CHAM Rules**

\[
A \vdash^L C[\{(p|q)^T\}] \equiv_P A \vdash^L C[\{p^T, q^T\}]
\]

\[
A \vdash^L C[\{(p|q)^T\}] \equiv_C A + \{a, \overline{a}\} \vdash^L C[\{p^{T+\{a\}}, q^{T+\{\overline{a}\}}\}] \quad \text{if} \{a, \overline{a}\} \cap (A + L) = \emptyset
\]

\[
A \vdash^L C[\{p^T\}] \rightarrow_{T_L} A \vdash^L C[\{\}]
\]

\[
A + \{a\} \vdash^L S \rightarrow_{S_L} A \vdash^L S
\]

\[
A \vdash^L C[\{(\alpha.p)^T_1, (\overline{\alpha}.q)^T_2\}] \rightarrow_R A - \left(\overline{T_1 \oplus T_2}\right) \vdash^L -\left(\overline{T_1 \oplus T_2}\right) C[\{p^\emptyset, q^\emptyset\}]
\]

if \(T_1 \cup T_2 \subseteq A + L\) and \(T_1 \cap \overline{T_2} = \emptyset\)
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Results

Label Transition relation $\leftrightarrow^\alpha$ defined as:

$$A \vdash^L C[\{(\alpha.p)^T\}] \leftrightarrow^\alpha A \vdash^{\overline{T}} \vdash^L \overline{T} \vdash^{\overline{T}} C[\{p^0\}]$$

if $T \subseteq A + L$ and $C[\ ]$ does not restrict $\alpha$.

Translation

$$\langle p \rangle = \emptyset \vdash^\emptyset \{p^0\}$$

- \text{TCCS CHAM satisfies standardization}
- \text{TCCS CHAM is fully abstract w.r.t. standard LTS semantics}
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**Sccs**

\( \text{Act} \) — Abelian monoid of actions under \( \cdot \). Assume an annihilator action \( 0 \).
\( \alpha \in \text{Act} \), and \( X \subseteq \text{Act} \).

**Processes**

\[
p ::= 0 \mid \alpha.p \mid p_1 \parallel p_2 \mid \nu X . p \mid p_1 \parallel p_2 \mid \text{fix}_i (\vec{x} = \vec{p})
\]

**Operational semantics**

\[
\frac{\alpha.p \rightarrow^\alpha p}{p_1 \parallel p_2 \rightarrow^\alpha p_i}
\]

\[
\frac{p_1 \rightarrow^\alpha p_1' \quad p_2 \rightarrow^\alpha p_2'}{p_1 \parallel p_2 \rightarrow^\alpha p_1' \parallel p_2'}
\]

\[
\frac{p_i \rightarrow^\alpha p_i'}{p_1 \parallel p_2 \rightarrow^\alpha p_i'} \quad (i = 1 \text{ or } 2)
\]

\[
\frac{\nu X . p \rightarrow^\alpha \nu X . p'}{\alpha \not\in X}
\]

\[
\frac{(p_i[\text{fix}_j (\vec{x} = \vec{p}) / x_j]_{j=1}^n) \rightarrow^\alpha p'}{\text{fix}_i (\vec{x} = \vec{p}) \rightarrow^\alpha p'}
\]
A CHAM for SCCS

Alternative Chemical Law?

\[
\begin{align*}
& S_1 \rightarrow S'_1 \\
\frac{S_1 \cup S_2}{S_2 \rightarrow S'_2}
\end{align*}
\]

But this only applies to reactions, not administrative heating/clean-up.

Context problem: Synchronization at an upper level can determine whether or not an action can occur. Consider \( \nu\{xy\}.(x.p|y.q) \) or \( (x.p|y.q)|0 \)

\( xy \) action not possible at the top level (prohibited or annihilated).

In SOS, can see what potential actions are realizable (global view).

But rewriting machinery of CHAMs is local, only goes forwards.

Want CHAM that works asynchronously in solutions, with coordination only while propagating possible actions.
**SCCS CHAM tags, molecules**

Labels $\mathcal{L} = \{a, b, c, \ldots\}$ (activity markers). $l, l_1, l_2, \ldots \subseteq_{fin} \mathcal{L}$.

Tags $t ::= \circ | \ell | \sum_{i=1}^{n} (\alpha_i, l_i)$

- $\circ$ — molecule ready to ionize, heat, propagate tag upwards.
- $\ell$ — “waiting tag”. Molecule’s fate pending. Invariant: tags on elementary particles kept non-intersecting.
- $\sum_{i=1}^{n} (\alpha_i, l_i)$ — superposed action choices tag. Abelian monoid structure to these as well.

**Molecules:** 2 kinds: $m_p$ are par-soluble, $m_c$ choice-soluble.

$$m_p ::= p^t \mid (\nu X. S_1^{t_1})^{t_2} \mid S_c^t \quad m_c ::= p^t \mid (\nu X. S_1^{t_1})^{t_2} \mid S^t$$

**Solutions:** Also two kinds. $S$ are p-solutions, and $S_c$ are c-solutions. Different membranes $\{ | \ldots | \}$ and $\langle \ldots \rangle$ (immiscible solvents).
Some SCCS CHAM rules

Some Heating cooling rules

\[ A \vdash \langle (\alpha.p)^\circ \rangle \iff I \quad A \vdash \langle (\alpha,0) \rangle \]

\[ A \vdash \{ (p|q)^\circ \} \iff PP \quad A \vdash \{ p^\circ, q^\circ \} \]

\[ A \vdash \{ (p\parallel q)^\circ \} \iff PC \quad A \vdash \{ \langle p^\circ, q^\circ \rangle \} \]

\[ A \vdash \langle (p|q)^\circ \rangle \iff CP \quad A \vdash \{ \{ p^\circ, q^\circ \}\}^\circ \]

\[ A \vdash \langle (p\parallel q)^\circ \rangle \iff CC \quad A \vdash \langle p^\circ, q^\circ \rangle \]

\[ A \vdash \langle 0^\circ \rangle \iff_0 A \vdash \langle 0^{(0,0)} \rangle \]
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Some tag Propagation rules

\[ A \vdash \langle M_1^{\beta_1}, \ldots, M_n^{\beta_n}, M_{n+1}^{(0, \emptyset)}, \ldots, M_k^{(0, \emptyset)} \rangle^\circ \rightarrow_{EU} \]

\[ A + \{a_1, \ldots, a_n\} - \bigcup_{i=n+1}^k \{L(M_i)\} \vdash \langle M_1^{\{a_1\}}, \ldots, M_n^{\{a_n\}} \rangle \sum_{i=1}^n (\{a_i\} \triangleright \beta_i) \]

\[ A \vdash \{M_1^{\beta_1}, \ldots, M_n^{\beta_n}\}^\circ \leftarrow_{SU} \]

\[ A \vdash \{M_1^{\circ}, \ldots, M_n^{\circ}\}^{\beta_1 \cdots \beta_n} \]

\( EU \) — “Superposition” and marking in external choice

\( SU \) — Upward propagation in parallel composition.

Alternative Chemical Law possible for these Propagation rules.
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Reaction and Clean-up Rules

**Reaction** is a *top level labelled rewrite*.

\[
A \vdash S\sum_{i=1}^{n} (\alpha_i, \ell_i) \xrightarrow{\alpha_j} R A - (\cup_{k=1, k \neq j}^{n} \ell_k) - \ell_j \vdash S^c
\]

Clean-up Rules

- \[ A \vdash \{ M^c \} \xrightarrow{CD} A - L(M^c) \vdash \{ \} \quad \text{if} \ l \not\subseteq A \]
- \[ A \vdash \{ \{ p^\ell \}^\circ \} \xrightarrow{CS} A - \ell \vdash \{ p^\circ \} \quad \text{if} \ l \subseteq A \]
- \[ A \vdash \{ \{ (\nu X . S^t)^\ell \}^\circ \} \xrightarrow{CS} A - \ell \vdash \{ (\nu X . S^t)^\circ \} \quad \text{if} \ l \subseteq A \]
- \[ A \vdash \{ \{ S^\ell \}^\circ \} \xrightarrow{CS} A - \ell \vdash S \quad \text{if} \ l \subseteq A \]

- \((CD)\) — discard choice branch.
- \((CS)\) — select in singleton choice, retire labels and remove choice membrane.
Results

Administrative moves:

\[ \rightarrow \mathcal{A} = \rightarrow \cup \sim \cup \leftarrow \]

Translation:

\[ \langle p \rangle = \emptyset \vdash \{ p^\circ \}^\circ \]

- SCCS CHAM satisfies standardization
- SCCS CHAM is fully abstract w.r.t. standard LTS semantics
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The Home Stretch
The Essence of CHAMS

Essentially Rewriting modulo AC. Rewrite steps which have an equational interpretation are heating or clean-up. Non-confluent rewrites = Reactions.

Then overlay a control catalysis – different ones for synchrony and asynchrony.

- The structural rules are factored into a “core” AC equality theory and an orientable set of rewrite rules.
- Restriction can also be treated as “somehow” AC
- Commutation of rules: $\rightarrow_A$ confluence and strongly coherent with $\rightarrow$.
- Formulation ensures coherence by avoiding problematic critical pairs.
Morals of the story

- Confluence-Coherence very useful properties
- Locality is essentially about these, not about Chemical Law formulation.
- Structural Laws arise from Commutation
- Good CHAM formulations try to simplify establishing coherence.
Things to do

- Refinements of a “chemistry”: Implement abstract chemistry in a more detailed chemistry. Weaker coherence properties to establish “correctness”.
- SCCS and TCCS choice extremes of a “bounded waiting” spectrum. Other choices? Encodings?
- Milner: ccs encoded in SCCS. Is there an abstraction relating SCCS CHAM and the TCCS CHAM?
Thank You!

Questions?